# THE DISPERSION AND DEPOSITION OF AEROSOLS<sup>1</sup>

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The dispersion and deposition of aerosols is considered on the basis of Sutton's theory for the diffusion of gases in the lower atmosphere and Sell's work on the efficiency of deposition of particles by impaction. An estimate is made of the rate of deposition of particles dispersed from a line source under various meteorological conditions. Application of the impaction curves has been made to the penetration and deposition of insecticidal aerosols through a forest canopy. Under conditions of low wind and in the absence of a downdraft, the theory predicts that the larger particles will penetrate the canopy better than the small. The opposite is true, however, when the dissemination depends on the downdraft from a plane. Sell's curves may also be used to predict the effect of the drop size of aerosols on the mortality rate of moving and stationary insects. The calculated results agree closely with the experimental observations of La Mer and coworkers.

The dissemination of particles suspended in the atmosphere is a problem of great practical importance. The dispersal of smokes and fumes from industrial plants is of concern to public health authorities and has received much attention (2, 8). The mathematical theory of atmospheric diffusion and its application to military problems in chemical warfare have been discussed in a recent paper by Sutton (8). Insecticides are often dispersed in the form of aerosols of small solid or liquid particles. For maximum efficiency these must be deposited uniformly on the ground or water surfaces, or on vegetation inhabited by the insects to be destroyed. Occasionally it is desirable to combat insects while they themselves are flying. Direct contact with aerosol particles by impaction on the wing or body then becomes important. Optimum results are obtained by dispersing the aerosol in particles containing approximately the lethal amount of the insecticide. An examination of some of the fundamental concepts of aerosols has proven valuable when interpreting results obtained in the laboratory and field, and in predicting the effects to be obtained under various meteorological and topographical conditions.

## DEPOSITION IN OPEN AREAS

When an aerosol is dispersed from a generator the particles are carried by the wind with little tendency to settle. It can be shown readily that the rate of fall of particles, even as large as 50 microns diameter, is small compared to the normal turbulent diffusion processes of the atmosphere. Deposition on the

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ground takes place therefore, not by settling through the height of the cloud, as has frequently been supposed, but by the particles actually being brought to the ground, just as the molecules of a gas cloud are mixed and brought to the ground by eddy diffusion. Adjacent to the ground is a more or less stagnant layer through which aerosol particles must then pass by true settling, or by Brownian diffusion. The latter may be neglected for particles of diameter larger than 0.5 micron. Thus not all of the particles that come close to the ground are deposited. Some are carried aloft again by wind currents. The rate of deposition of the aerosol particles per unit area is obviously the product of the concentration at ground level and the settling velocity.

The problem of diffusion of gases in the lower atmosphere has been treated extensively in the recent papers by Sutton (7, 8). While the theoretical expressions cannot be claimed to give highly accurate values for concentrations of gases released into the air because of the widely varying conditions in the winds, they are reasonable guides when observations on the wind gradient are made above the layer in which the effects of surface roughness predominate. In order to use Sutton's equations for aerosols which settle out, a correction must be made for the amount of material which is deposited. For particles larger than 1 micron this correction becomes important. The following derivation applies to a cross wind line source but analogous considerations may be applied to a continuous point source.

The equation for the downwind concentration of a gas at a distance x from a line source of strength W is<sup>3</sup>

$$C = \frac{W e^{-z^2/c_{z}^2 x^m}}{\sqrt{\pi} c_z \, u x^{m/2}} \tag{1}$$

- <sup>3</sup> Nomenclature: consistent units are used except as indicated in the text.
- $c_{s}$  = vertical diffusion coefficient depending on meteorological conditions
- C =concentration of cloud
- Ct = concentration-time product; subscript g refers to gas cloud, a to aerosol cloud
- D = dimension of impacting object, diameter of sphere
- d = diameter of particle
- f = fraction of particles remaining airborne
- g =acceleration of gravity
- h = height of source above ground
- k = proportionality factor relating drag resistance to velocity of particle
- M = lethal amount of insecticide
- m = a meteorological constant
- n = number of units of distance; also number of particles of insecticide containing lethal dose
- Q = number of particles crossing vertical plane of unit area
- t = time
- u =mean wind velocity
- v = velocity of downdraft
- $v_s$  = rate of settling of particle

W = line source strength, mass of agent emitted per unit time per unit distance

- $W_{n^+}$  = probability that n or more particles will strike a given area in time t
  - w = mass of particle
  - x = horizontal distance

The quantity  $c_s$  is a generalized diffusion coefficient which depends on meteorological conditions, the gustiness of the atmosphere, and only slightly on u, the mean wind speed. Gustiness is obviously a variable quantity and depends upon many things, including the net radiation received by the ground, the topography, and the height of observation. The exponent m is also a meteorological constant which may be measured by the wind profile. Its value varies from approximately 1.85, for conditions of high turbulence due to a large lapse rate, to about 1.4 for inversion conditions such as frequently occur in the late evenings or at dawn. Equation 1 does not take into account the effect of the ground on the gas concentration. Allowing for the reflection of the cloud at the plane z = 0, when the source is at the height h, the equation becomes

$$C = \frac{W}{\sqrt{\pi} c_z u x^{m/2}} \left[ e^{-(z-h)^2/c_z^2 x^m} + e^{-(z+h)^2/c_z^2 x^m} \right]$$
(2)

Now in an aerosol cloud in which deposition occurs by settling across the stagnant layer adjacent to the ground, reflection is not complete. Furthermore, if the particles are large there is some tendency for settling to take place even in the turbulent region. Introducing these corrections the equation becomes

$$C = \frac{W}{\sqrt{\pi} c_z u x^{m/2}} \left[ e^{-\left(z + \frac{v_z x}{u} - h\right)^2 / c_z^2 x^m} + \alpha e^{-\left(z + \frac{v_z x}{u} + h\right)^2 / c_z^2 x^m} \right]$$
(3)

Here  $\alpha$  is the average reflection coefficient up to the distance x; that is, it is that fraction of the material in the cloud brought to the ground that is not deposited.

The average value of  $\alpha$  may be determined by the material balance:

$$\frac{1}{W} \int_0^x C_0 v_s \, \mathrm{d}x = 1 - \frac{1}{W} \int_0^\infty C u \, \mathrm{d}z \tag{4}$$

Equation 4 states that the fraction deposited is the fraction that does not diffuse across an infinite vertical plane at the distance x from the source. If the line source is at ground level, and if the effect of settling in the turbulent region can be neglected, equation 4 gives

$$\frac{(1+\alpha)v_s}{\sqrt{\pi}\,c_z\,u}\,\int_0^x\frac{\mathrm{d}x}{x^{m/2}}\,=\,1\,-\,\frac{(1+\alpha)}{\sqrt{\pi}\,c_z\,x^{m/2}}\int_0^\infty e^{-z^2/x^m c_z^2}\,\mathrm{d}x\tag{5}$$

- $\delta$ ,  $\gamma$  = characterization constants for foliage
  - e = efficiency of deposition of aerosol particles on objects by impaction

- $\varphi$  = average number of particles striking given surface per unit time
- $\sigma$  = surface area of insect for deposition of aerosol particles; subscript h refers to resting insect; f to flying insect; v to frontal impact area of flying insect
- $\rho$  = density of aerosol particles

z = vertical distance

 $<sup>\</sup>alpha$  = reflection coefficient fraction of aerosol cloud brought to the ground by turbulent diffusion that is not deposited

 $<sup>\</sup>mu$  = viscosity of air

or

$$\frac{(1+\alpha)v_s}{\sqrt{\pi}\,c_s\,u}\,\frac{x^{1-m/2}}{\left(1-\frac{m}{2}\right)}\,=\,1\,-\,\frac{1+\alpha}{2}\tag{6}$$

Either side of equation 6 expresses the fraction deposited as a function of distance. Solving for the value of the reflection coefficient,

$$\alpha = \frac{2}{1 + \frac{2v_s x^{1-m/2}}{\sqrt{\pi} u \left(1 - \frac{m}{2}\right) c_s}} - 1$$
(7)

## TABLE 1

Fraction of homogeneous aerosol clouds deposited at various distances from a continuous line source on the ground

| DISTANCE | WIND VELOCITY = 2 MILES PER HOUR Particle diameter in microns |       |      |      |      | WIND VELOCITY = 5 MILES FER HOUR Particle diameter in microns |      |      |      |      |
|----------|---|-------|------|------|------|---|------|------|------|------|
|          |   |       |      |      |      |   |      |      |      |      |
|          | yards   | _     |      | 1    |      | 1   |      |      |      |      |
| 10       | 0.005   | 0.129 | 0.37 | 0.70 | 0.90 | 0.000   | 0.06 | 0.18 | 0.49 | 0.85 |
| 50       | 0.007   | 0.154 | 0.42 | 0.74 | 0.93 | 0.002   | 0.07 | 0.22 | 0.54 | 0.88 |
| 100      | 0.009   | 0.163 | 0.44 | 0.76 | 0.95 | 0.002   | 0.07 | 0.24 | 0.57 | 0.88 |
| 500      | 0.010   | 0.195 | 0.49 | 0.79 | 0.96 | 0.004   | 0.09 | 0.28 | 0.61 | 0.90 |
| 1000     | 0.012   | 0.208 | 0.51 | 0.81 | 0.96 | 0.004   | 0.10 | 0.30 | 0.63 | 0.91 |
| 5000     | 0.014   | 0.245 | 0.56 | 0.84 | 0.97 | 0.005   | 0.11 | 0.34 | 0.68 | 0.93 |
| 10000    | 0.015   | 0.263 | 0.58 | 0.85 | 0.97 | 0.006   | 0.12 | 0.36 | 0.70 | 0.93 |

In order to compare the amount of deposition from aerosol clouds dispersed over an open terrain, solution of equation 6 has been made using values of  $c_s = 0.12$  and m = 1.75 which are given by Sutton (8) for "average" conditions, corresponding to a small vertical temperature gradient and a low wind velocity. The results of the calculations are shown in table 1. It is interesting to note that, under the conditions chosen, most of the deposition takes place near the source. This is because the ground concentration in this area is very high. Aerosols with particles less than 10 microns in diameter, however, are carried for considerable distances without serious depletion of the concentration by deposition. The normal dilution of the cloud by vertical expansion decreases the ground concentration so much that little deposition occurs beyond 100 yards from the source. It should be noted that, if the source were located slightly above the ground and for smaller values of m and  $c_z$  corresponding to inversion conditions, more nearly uniform deposition can be obtained for short distances from the generator before the cloud is depleted.

### HORIZONTAL PENETRATION OF AEROSOLS THROUGH FOLIAGE

It is frequently of interest to estimate how far an aerosol will penetrate through foliage, such as a forested area or low vegetation. Of course, the problem cannot be solved completely without knowing the factors that char-

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acterize the size and shape of the obstacles in the path of the aerosol. Certain generalities can be reached, however, by characterizing the denseness of the foliage in terms of vertical and horizontal lengths. It is necessary to consider the efficiency of impaction of small particles on the surfaces, as it is well known that aerosol particles tend to follow the streamlines of the gas and thus to be carried around the surfaces of obstacles placed in their path. Some interesting conclusions concerning the efficiency of various shapes of bodies for collecting dust particles from gas streams have been reached by Sell (6), from a simple analysis of the motion of dust particles. From experimental measurements on the streamlines about bodies he determined the efficiency of deposition of

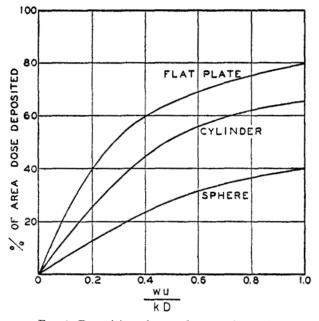


FIG. 1. Deposition of aerosols on surfaces (6)

aerosol particles,  $\epsilon$ , as a function of the group wu/kD. For small spherical particles in the Stokes' law range

$$k = 3\pi\mu d \tag{8}$$

and

$$\frac{wu}{kD} = \frac{1}{18} \frac{d^2 u\rho}{\mu D} \tag{9}$$

The efficiency of deposition of aerosol particles on circular cylinders, discs, and spheres is shown in figure 1.

The density of foliage may be characterized by two lengths,  $\delta$  and  $\gamma$ , where  $\delta$  is the horizontal distance for which the sum of the vertical foliage surfaces in any cross section is equal to the cross section, and  $\gamma$  is a similar distance in the vertical direction. The ratio  $\delta/\gamma$  will be taken, for the purpose of illustration,

as equal to 2; that is, the foliage will be considered as twice as dense in the vertical direction as in a horizontal direction. This seems to be reasonable in view of the fact that a larger proportion of the leaf surfaces are horizontal. When an aerosol penetrates horizontally through the foliage, the particles will be deposited in two ways, by impingement on vertical surfaces and by settling on horizontal surfaces. The fraction of the total number of particles lost in traveling the horizontal distance  $\Delta x$  is given by

$$-\frac{\Delta Q}{Q} = \frac{\epsilon \Delta x}{\delta} + \frac{\Delta z}{\gamma} \tag{10}$$

where  $\Delta z$  is the distance fallen in a vertical direction, and  $\epsilon$  is the fraction of the area dose deposited on a vertical surface as given by Sell. We shall assume that  $\epsilon$  lies between the value for a flat plate and for a circular cylinder. Of course, D will vary with the type of foliage but for purposes of illustration let us assume that it is 1 cm. If  $v_s$  is the settling velocity of the drops, we may write the above equation as

$$-\frac{\Delta Q}{Q} = \frac{\epsilon \Delta x}{\delta} + \frac{v_s}{u} \frac{\Delta z}{\gamma}$$
(11)

If we let n be the number of units of  $\delta$  traveled downwind, that is, the number of times that a small shot on the average would penetrate through a surface in traveling the distance  $x, n = x/\delta$ , we have

$$-\frac{\mathrm{d}Q}{Q} = \epsilon \,\mathrm{d}n + \frac{v_s}{u}\frac{\delta}{\gamma}\,\mathrm{d}n\tag{12}$$

The solution of this is  $Q = fQ_0$ , where f is the fraction penetrating to a distance n downwind, and

$$f = e^{-(\epsilon + (v_{\epsilon}/u)(\delta/\gamma))n}$$
(13)

The quantity f has been evaluated as a function of n for various size drops and for wind speeds of 5 and 1 miles per hour. The results are shown in figure 2. In using units of n, no particular denseness of the foliage has been assumed. Reasonable values of  $\delta$  for dense and light foliage are perhaps 5 and 100 ft., respectively.

### VERTICAL PENETRATION

A similar consideration of the vertical penetration of an aerosol through foliage leads to some interesting conclusions concerning the dispersion of aerosols over a forested area by means of an airplane, especially when the effect of the downdraft from the plane is taken into consideration. Using the same notation as in the discussion on horizontal penetration, if the aerosol is settling downward without any vertical component of the wind,

$$-\frac{\mathrm{d}Q}{Q} = \frac{\epsilon u}{v_s} \,\mathrm{d}n \,+\, 2 \,\mathrm{d}n \tag{14}$$

$$f = e^{-(2 + \epsilon u/v_s)n} \tag{15}$$

The ratio  $\delta/\gamma$  is again assumed to be 2 and *n* is the number of units of the horizontal distance  $\delta$  measured from the top of the canopy. The fraction *f* of the aerosol penetrating to a depth *n* is shown in figure 3 for drops of various sizes, and for a crosswind through the foliage of 1 mile per hour. If there is no cross-

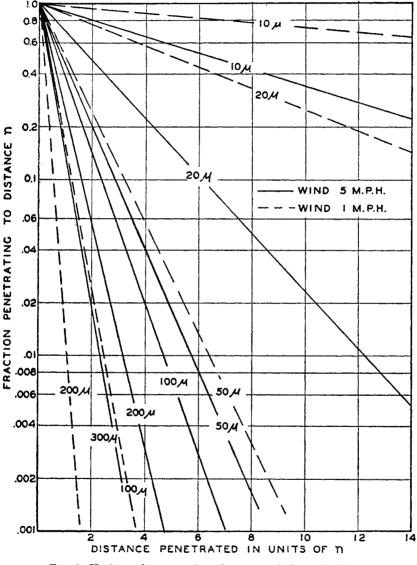


FIG. 2. Horizontal penetration of an aerosol through a forest

wind, drops of all sizes will be deposited on the foliage to the same extent. This is shown by the broken line in the figure. It thus appears that when there is a crosswind and no downdraft, the larger drops will penetrate downward through a forested area more efficiently than the smaller drops. This apparent anomaly can be explained by examining the effect of the diameter of the particle on the factors in the exponent of equation 15. The diameter of a particle affects both the efficiency of deposition  $(\epsilon)$  and the rate of fall  $(v_s)$ . For diameters less than

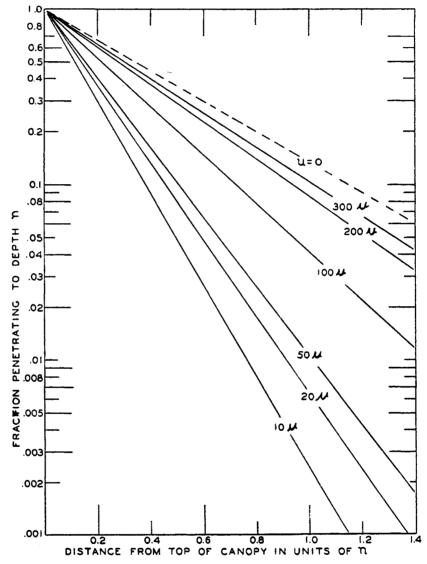


FIG. 3. Vertical penetration of an aerosol through foliage; crosswind velocity 1 mile per hour; no downdraft.

80 microns, both numerator and denominator are approximately proportional to  $d^2$ . For larger particles, both quantities increase more slowly than  $d^2$ , the efficiency leveling off at a smaller value of d for any value of D than does the settling rate  $v_s$ , which continues to increase with the diameter. This tends to

make the value of f larger for the larger drops. Physically, the effect is essentially that of the larger drops falling faster and not having as much chance to impinge on the foliage in a horizontal wind as do the smaller drops, since they are not carried past as many vertical surfaces.

If the aerosol is dispersed by an airplane which is flying only a short distance above the foliage, the downwash of the plane will drive the aerosol downward through the layers of foliage. Since the impaction on the foliage is a function of the square of the drop size, this downward component will increase the relative penetration efficiency of the smaller drops. If there is a downwash from the plane it is reasonable to assume that it dies out fairly rapidly in the foliage and that the time rate of decrease is proportional to the square of the velocity, i.e.,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv^2 \tag{16}$$

where v is the downwash velocity and k is a constant of proportionality. This leads to

$$v = v_0 e^{-kz} \tag{17}$$

where  $v_0$  = the downwash velocity at the top of the foliage canopy and v is the downwash velocity at a distance z below the top. k has the dimensions of reciprocal distance. We shall arbitrarily take  $k = 1/\gamma$ , so that we have

$$v = v_0 e^{-2n}$$
 (18)

where n is again the number of units of  $\delta$  below the top of the canopy.

The fraction of aerosol lost in penetrating a distance dy is

$$-\frac{\mathrm{d}Q}{Q} = \epsilon(u)\frac{\mathrm{d}x}{\delta} + [f\{\epsilon(v), V_s\}]\frac{\mathrm{d}z}{\gamma}$$
(19)

The quantity in brackets refers to the combined fractions deposited due to settling and impingement. This quantity is not known and no attempt is made to evaluate it. We shall make the assumption that, when  $v > V_s$ , deposition takes place only by impingement, and, when  $v < V_s$ , deposition takes place only by impingement, and when  $v < V_s$ , deposition takes place only by settling. This approximation is admittedly rough, but it will give an indication of the true state of affairs. Also, since the form of  $\epsilon(u)$  is not known and v contains y explicitly, we cannot integrate the above equation directly, so we define an average velocity  $\bar{v}$  where

$$\bar{v} = \frac{\int_{n_1}^{n_2} v_0 \, \epsilon^{-2n} \, \mathrm{d}n}{n_2 - n_1} \tag{20}$$

or

$$\bar{v} = \frac{v_0}{2(n_2 - n_1)} \left( e^{-2n_1} - e^{-2n_2} \right)$$
(21)

and calculate the penetration step by step, using

$$f_i = f_{i-1} e^{-\left(\epsilon(\bar{v})\frac{\delta}{\bar{\eta}} + \epsilon(u)\frac{u}{\bar{v}}\right)^{(n_i - n_{i-1})}} \qquad \text{for } \bar{v} > V_s \tag{22}$$

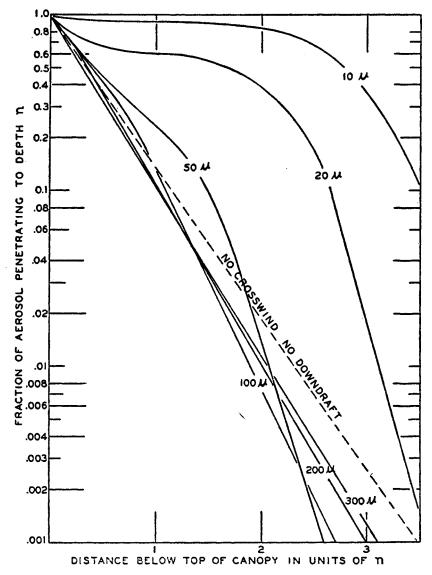


FIG. 4. Vertical penetration of an aerosol through foliage in the presence of a downdraft. u, wind velocity, 1 mile per hour;  $v_o$ , downdraft, 5 miles per hour.

and

$$f = f_k e^{-\left(\frac{\delta}{\gamma} + \epsilon(u)\frac{u}{\bar{v}_s}\right)(n - n_k)} \qquad \text{for } \bar{v} < V_s$$
(23)

where  $f_k$  is the value of f at the point where  $\bar{v} = V_s$ .

The ratio f has been evaluated as a function of n, using increments of n = 0.2 for calculation. The results for various drop sizes are shown in figure 4. The solid lines are for a crosswind of 1 mile per hour and a downdraft of  $v_0 = 5$ 

miles per hour. The dashed line shows the penetration for  $v_0 = 0$  and u = 0. The effect of the downwash in increasing the penetration efficiency of the smaller drops is quite evident. After a certain depth has been reached, the downdraft disappears and the drops deposit only by settling. Here the larger drops become more efficient in penetration.

The optimum position of a spray outlet on an airplane to get maximum penetration into foliage is the point where the downdraft is utilized to best advantage and the smallest percentage of the droplets are carried into the turbulent wake. This position should be forward of and below the trailing edge of the wing where the flow lines have maximum divergence. If the spray is too close to the trailing edge or to the lower surface of the airfoil, it will enter the wake and be dissipated in turbulent motion. If the spray is too far below the airfoil, the downward component of velocity may be quite small.

In the discussion above, no consideration has been given to inhomogeneities, or striations, which are always present in any natural foliage. Also, any updrafts which may be present due to meteorological turbulence are neglected. The vertical components of turbulence occurring in the lower layer of the atmosphere will keep some of the smaller drops from reaching ground level. This may be an advantage in forested regions by giving a more uniform deposit on the foliage, thus leading to better control by the residual effect of the insecticide.

## OPTIMUM DROP SIZE FOR CONTACT EFFECT OF INSECTICIDES

The effect of the particle size of the aerosol on the probability of hitting the insects with sufficient material to cause mortality is of interest. It is apparent that an optimum particle size must exist for each percentage kill. The application of statistical analysis to this problem was first suggested by Rodebush (5). The following treatment differs somewhat from the original and leads to some more general conclusions.

Consider the particles of an aerosol cloud falling upon a flat horizontal surface through still air. The velocity of fall depends upon the drop diameter. Consequently, different amounts will be deposited on any surface exposed to the same concentration-time product  $Ct_p$ , depending upon the diameter of the drops composing the aerosol cloud. Although each small area receives, on the average, a certain number of drops, there is a certain probability that it will receive more or less than this number. For instance, if the probability is 1/2that of the number of drops falling on each insect is equal to or more than the number of drops necessary to cause mortality, then 50 per cent of the insects so exposed will be killed.

Three general cases will be considered:

I. The mosquito<sup>4</sup> is stationary and the aerosol is deposited only by Stokes' law of settling of the drops.

II. The mosquito is flying horizontally at a speed of 3 miles per hour, and in

<sup>&</sup>lt;sup>4</sup> The discussion here is confined to mosquitoes but should be applicable to other insects as well.

addition to the drops that settle on it, some are picked up by impingement. Impingement on the wings, which is very likely an important factor due to their rapid motion, is neglected.

III. The mosquito is resting on a screen through which the aerosol is passing with the wind at different velocities.

In the numerical calculations, the lethal amount of the insecticide must be known. For DDT, the lethal value for mosquitoes is between  $10^{-8}$  and  $10^{-10}$  g.

## Case I. Resting mosquito

The average mass of aerosol deposited in unit time on a horizontal surface of area  $\sigma_h$  is given by

$$Cu\sigma_h = C \frac{d^2 \sigma_h g}{18\mu} \tag{24}$$

The average number of drops deposited on  $\sigma_h$  per unit time is

$$\varphi = \frac{C}{3} \frac{\sigma_h}{\pi} \frac{g}{\mu d} \tag{25}$$

The cloud concentration necessary to give this number is

$$C = \frac{3\pi\mu d\varphi}{g\sigma_h} \tag{26}$$

Then the value of  $Ct_p$  necessary to have an average of  $\varphi t$  drops strike the area  $\sigma_h$  in time t is

$$Ct_p = \frac{3\pi\mu d}{g\sigma_h} \cdot \varphi t \tag{27}$$

For any given value of M, the number of drops, n, of a given size to kill a mosquito for any concentration of DDT in inert solvents is known. Now the problem is: What is the average number of drops,  $\varphi t$ , that must strike an area so that there is a certain probability  $W_{n+}$  that n or more drops strike any one area? This probability is the per cent mortality experienced by the mosquitoes when n drops contain the lethal amount M. Knowing the average  $\varphi t$  necessary, the value of  $Ct_p$  is found immediately from equation 27.

Probability considerations: Assuming that the dispersion of the drops is random, a solution can be obtained by the method of Bateman (1). The probability that one drop strikes in time dt is  $\varphi dt$ . Let  $W_{n+1}(t + dt)$  equal the probability that n + 1 drops strike in time t + dt. This can occur in two ways: n + 1 drops in time t and none in dt, or n drops in time t and 1 in dt. This gives:

$$W_{n+1}(t + dt) = (1 - dt)W_{n+1}(t) + (dt)W_n(t)$$
(28)

When dt approaches zero, we have:

$$\frac{\mathrm{d}W_{n+1}}{\mathrm{d}t} = (W_n - W_{n+1})$$
(29)

The general solution of this set of equations is

$$W_n = e^{-\varphi t} \frac{(\varphi t)^n}{n!} \tag{30}$$

This is simply the Poisson distribution equation.  $W_n$  is the probability that exactly *n* drops will strike in time *t* if the average number per unit time is  $\varphi$ . However, we want to know the probability that *n* or more drops will strike. Calling this probability  $W_{n+}$ , we have

$$W_n + = e^{-\varphi t} \sum_{k=n}^{k=\infty} \frac{(\varphi t)^k}{k!}$$

or,

$$W_{n^+} = 1 - e^{-\varphi t} \sum_{k=0}^{k=n} \frac{(\varphi t)^k}{k!}$$
(31)

Equation 31 must be solved for a given value of n, which depends on the toxicity, the drop size, and the concentration of insecticide in the drops, to find the value of  $\varphi t$  required to give each percentage kill. Three cases have been considered:  $W_{n^+} = 0.50, 0.90, \text{ and } 0.99.$ 

The calculations were carried out for small values of n by plotting  $W_{n+}$  vs.  $\varphi t$  for any given n and determining the value of  $\varphi t$  necessary to give a chosen  $W_{n+}$  graphically. For large values of n, i.e., for very small drops, the value of  $\varphi t$  approaches the value of n, that is, the drops deposit in an almost uniform film.

Expressing d in microns and  $Ct_p$  in mg.min./cu.m., and taking  $\sigma_h = 4.7 \times 10^{-2}$  sq. cm., the fundamental equation becomes

$$Ct_p = 6.16 \times 10^{-2} \, d\varphi t \, \mathrm{mg.min./cu.m.}$$
(32)

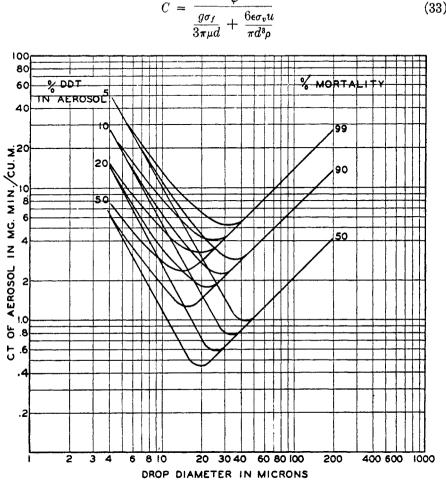
The results are shown in figure 5 for  $M = 10^{-9}$  g. The curves exhibit a definite optimum drop diameter for each per cent mortality. The initial steep decrease is due to the rapid increase in settling velocity with increasing drop size, leading to a slope of -2 for 50 per cent mortality. For larger drops and larger percentage kills, the efficiency becomes smaller, resulting in a lower slope. Finally, at approximately the drop size where one drop contains the lethal dose, the minimum is reached. For larger drops the number of drops necessary to kill can no longer decrease, and the curve increases linearly with d.

## Case II. Flying mosquito

For flying mosquitoes, in addition to the drops striking from above, there is a contribution to  $\varphi$  by impingement of the frontal area of the insect with the drops in the aerosol cloud. The fraction of drops which impact on the insect increases with drop size in accordance with Sell's theory. We then have

$$\varphi = \frac{gC\sigma_f}{3\pi\mu d} + \frac{6\epsilon C\sigma_v u}{\pi d^3\rho} \tag{32}$$

where u is now the speed of the insect through still air. The cloud concentration necessary to give this value of  $\varphi$  is



$$C = \frac{\varphi}{\frac{g\sigma_f}{3\pi\mu d} + \frac{6\epsilon\sigma_v u}{\pi d^3\rho}}$$
(33)

FIG. 5. Required Ct of aerosol for various mortality rates on flying mosquitoes. Lethal dose assumed to be 10<sup>-9</sup> g.

 $\epsilon$  is the fraction of the area dose deposited on the frontal area of the mosquito flying with a velocity u.

$$Ct_p = \frac{\varphi_t}{\frac{g\sigma_f}{3\pi\mu d} - \frac{6\epsilon\sigma_v u}{\pi d^s \rho}}$$
(34)

 $\epsilon$  is found from Sell's curves by assuming that we can represent the frontal area of the mosquito by a flat plate with an area of  $3 \times 10^{-2}$  sq. cm.  $\sigma_f$  is assumed to be  $10^{-1}$  sq. cm.

If d is expressed in microns and  $Ct_p$  in mg. min./cu.m.,

$$Ct_{p} = \frac{1.67\varphi td^{3}}{56.8d^{2} + 2780\epsilon}$$
(35)

The value of  $\varphi t$  was calculated as in case I. The results are shown in figure 6. The general shape of the curves is the same as in case I. There are two differences however. First, the  $Ct_p$  necessary to cause any percentage mortality in

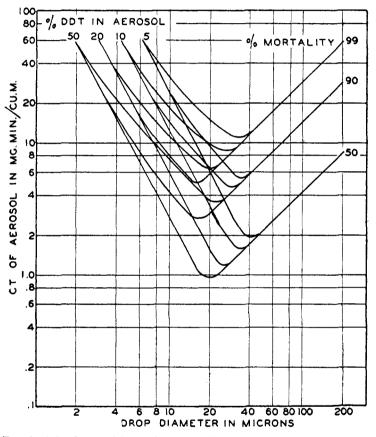


FIG. 6. Required Ct of aerosol for various mortality rates on resting mosquitoes. Lethal dose assumed to be  $10^{-9}$  g.

case II is approximately one-half that required in case I. This is mainly because the area  $\sigma_f$  is about twice the area  $\sigma_h$ . Secondly, the minima are shifted slightly to larger drop sizes in case II because of the fact that the  $\epsilon$  increases with the square of the drop diameter.

Confirmation of the shape of the curves for 50 per cent mortality and the general position of the minimum may be found in the experimental data of La Mer and Hochberg, who studied the efficiency of DDT aerosols for killing mosquitoes in static atmospheres in a closed chamber (3). The insects were

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exposed to aerosols of uniform drop sizes containing 8 per cent insecticide in lubricating oil. The results, expressed as a median lethal dose for female *Aedes aegypti*, are shown in figure 7. The parallel lines represent the calculated

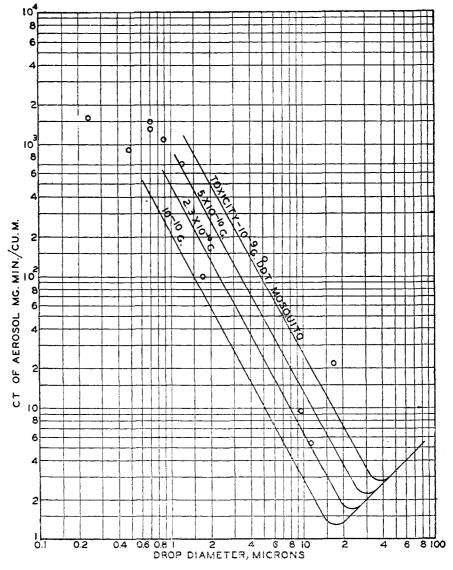


FIG. 7. Effect of particle size on dosage of DDT aerosol

effects for toxicities of  $10^{-9}$ ,  $5 \times 10^{-10}$ ,  $2.3 \times 10^{-10}$ , and  $10^{-10}$  g. of DDT per mosquito. The experimental data fall within this range. The calculated minimum dosage occurs at about 2 mg.min./cu.m. and at a drop diameter of about 30 microns for solutions of the given concentration. This is slightly

beyond the range of drop size at which the experiments were made. In experiments of this nature it is difficult to prevent residue effects entirely when the insects rest on the screen.

# Case III. Mosquitoes at rest on a screen in a moving air stream

Latta, La Mer, and coworkers have recently reported another series of experiments in which the effect of the drop size and wind velocity on the mortality

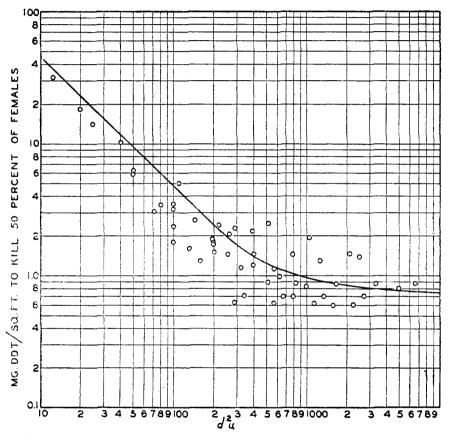


FIG. 8. Effect of particle size and velocity on amount of DDT aerosol required to produce 50 per cent mortality of mosquitoes. ———, calculated curve for toxic dose of  $2.4 \times 10^{-8}$  g. O, data of Latta *et al.* (J. Wash. Acad. Sci. **37**, 397 (1947)).

rate was determined when mosquitoes were placed in a screen cage in a wind tunnel (4). It is interesting to compare the experimental results with those calculated from the probability equation, taking into consideration the fact that the efficiency of impaction of the droplets on the mosquito is also a function of the drop diameter and wind velocity. The authors noted that even at low wind velocities the insects did not fly but clung to the forward screen of the cage and remained stationary until the velocity reached about 16 miles per hour, when the force was sufficient to blow them to the opposite screen. Under these conditions the deposition takes place mainly by impaction, and the settling of the particles on the relatively small horizontal area may be neglected. The amount of aerosol deposited in unit time then becomes

$$\varphi = \frac{6\epsilon C u \sigma_h}{\pi d^3 \rho} \tag{36}$$

The concentration-time product is

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$$Ct_p = \frac{\pi d^3 \rho \cdot \varphi t}{6\epsilon u \sigma_h} \tag{37}$$

The impaction efficiency  $\epsilon$  was calculated from the curve in figure 1 for a flat disk  $3 \times 10^{-2}$  sq. cm. in area.

From the values of  $\varphi t$  calculated from equation 31 for  $W_{n^+} = 0.5$  the value of the aerosol dose in milligrams of DDT per square foot was calculated for various values of  $d^2u$ . A value slightly lower than the median lethal value given by Latta and La Mer was used in these calculations: namely,  $M = 2.4 \times 10^{-8}$  g. The density of the droplets was taken as 1 g. per cubic centimeter and the viscosity of air at 20°C. was used. The results of the calculation are shown by the curve in figure 8. The experimental data of Latta and La Mer are shown by the points. The agreement between the predicted and experimental data is excellent.

### CONCLUSIONS

The agreement between Latta and La Mer's experimental results and those predicted from theoretical considerations lends support to the belief that the theory of aerosols may be applied to the dispersion of insecticides. Considerable discussion has taken place as to the relative merits of dispersing insecticide solutions as coarse drops and as aerosols of fine drops. There is theoretical and experimental evidence that small droplets are more effective than large drops when used under proper conditions. Practical difficulties of controlling the movement of aerosol clouds when dispersed from ground generators or from airplanes exist; yet, as the toxicity of insecticidal agents is increased, the advantages of using aerosols will also increase. The effectiveness of insecticidal aerosols dispersed from aircraft has been demonstrated in the field as a practical method of controlling natural insect populations. The problem remaining is that of working out the most effective technique of using the equipment on hand in order to secure the greatest economy of effort in use of the insecticide. Among the factors that are indicated in this study as important in aerial dispersion are such items as the type of aircraft, the position of the discharge on the plane, the speed of the plane, the height of the plane above the vegetation, the type of solvent used, the concentration of the solution, the quantity of solution dispersed, the drop spectrum and quantity of agent reaching the insect habitat, the type and height of vegetative cover, and the meteorological conditions.

It should be emphasized that neither theoretical studies nor laboratory experiments can be a complete substitute for field tests. Since the only object in the use of insecticides is to control insects, biological measurements must be the principal means of assessment of the results of any experiment. The importance of this has sometimes been overlooked in the eagerness of finding some physical measurement which could foretell the results. Good biological studies are admittedly difficult to make, especially when it is necessary to cover large areas and there is a natural fluctuation of the insect population due to weather conditions and breeding and flight habits. Careful correlation between the results on caged insects and natural population might eliminate some of this difficulty. The results on caged insects have often been misleading because of the deposition of the droplets on the screen and the slow rate at which equilibrium is obtained between the air in the cage and the conditions outside. Nevertheless, it should be possible to use caged insects to establish a definite correlation between biological results and the dosage and degree of contamination within the cage.

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